

Einstein's theory of relativity

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UCL Institute of Origins

Origins

UCL has established the Institute of Origins to promote world leading research in topics related to the Origins and Evolution of the Universe. Our research themes are:

- ▶ Galaxy Evolution
- ▶ Mathematical Foundations
- ▶ Neutrino physics
- ▶ Planetary Science

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Overview

- ▶ **Albert Einstein**
 - ▶ 1905 Einstein's "Wonderful Year"
 - ▶ 1915 General relativity
- ▶ Special relativity
 - ▶ the twin problem
 - ▶ mass-energy equivalence
- ▶ General relativity
 - ▶ matter and curvature
 - ▶ black holes and wormholes
- ▶ Cosmology
 - ▶ from the big bang to today
 - ▶ dark matter, dark energy and all that

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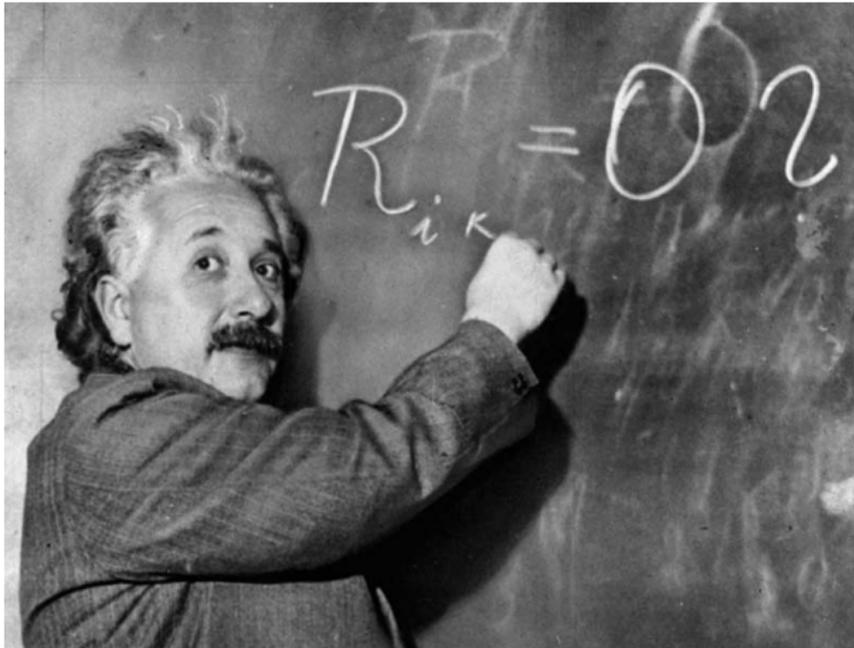
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Albert Einstein (1879 – 1955)



1905 – Annus Mirabilis

- ▶ particulate nature of light – photoelectric effect
- ▶ Brownian motion – confirming atomic theory
- ▶ electrodynamics of moving bodies – special relativity
- ▶ mass-energy equivalence – $E = mc^2$

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- ▶ for about 10 years Einstein worked on a generalisation of Newtonian gravity
- ▶ Marcel Grossmann and Tullio Levi-Civita influenced Einstein by discussing and introducing him to Riemannian geometry
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- ▶ $R_{ab} - \frac{1}{2}Rg_{ab} = 8\pi T_{ab}$

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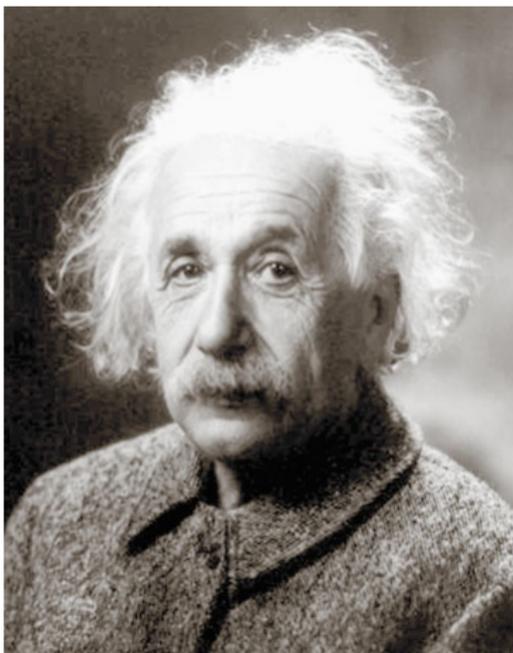
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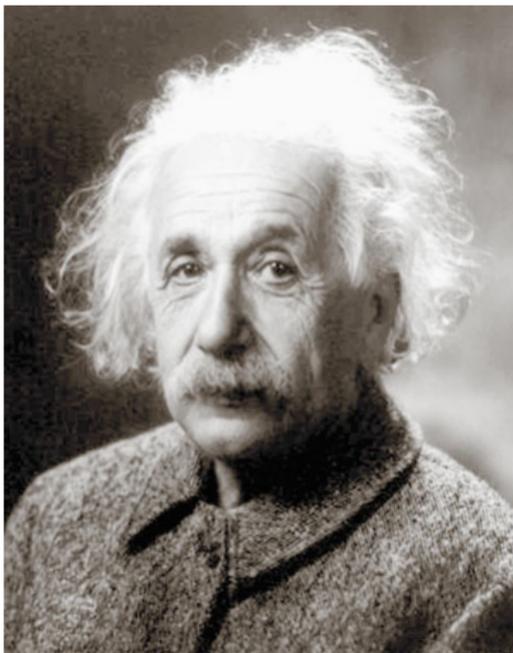
Albert Einstein 1947



Latter years

- ▶ Einstein refuses Quantum Theory
- ▶ subsequently refuses Quantum Field Theory
- ▶ “God does not play dice”

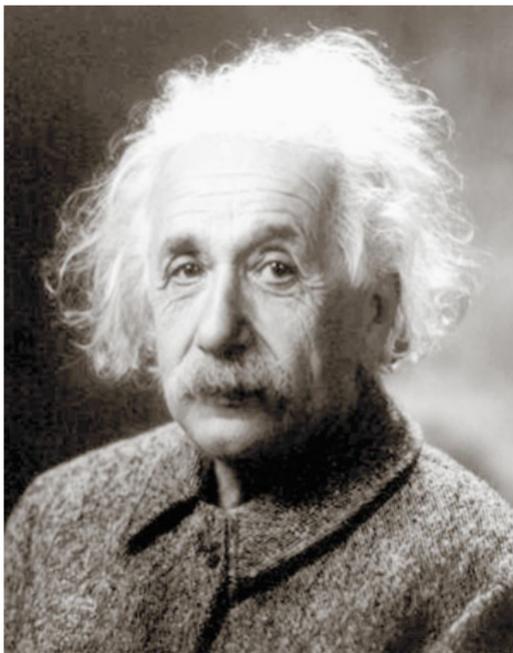
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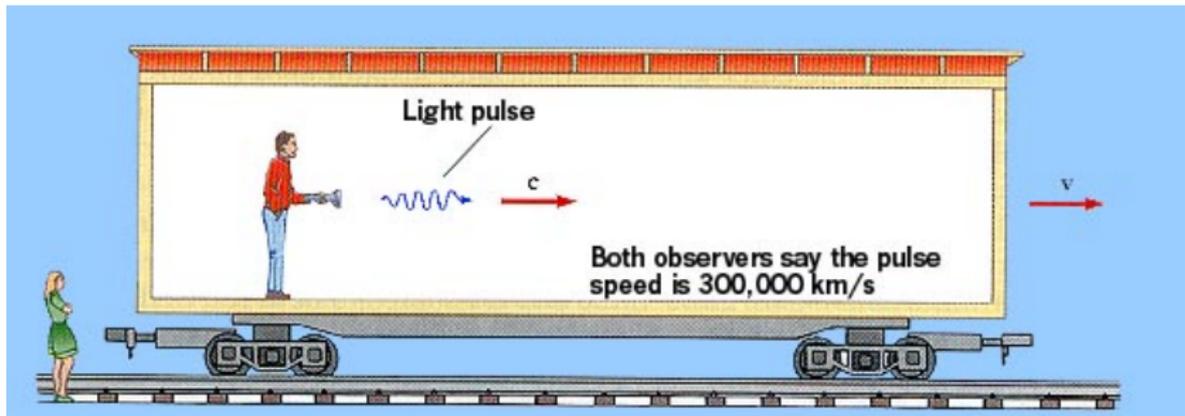
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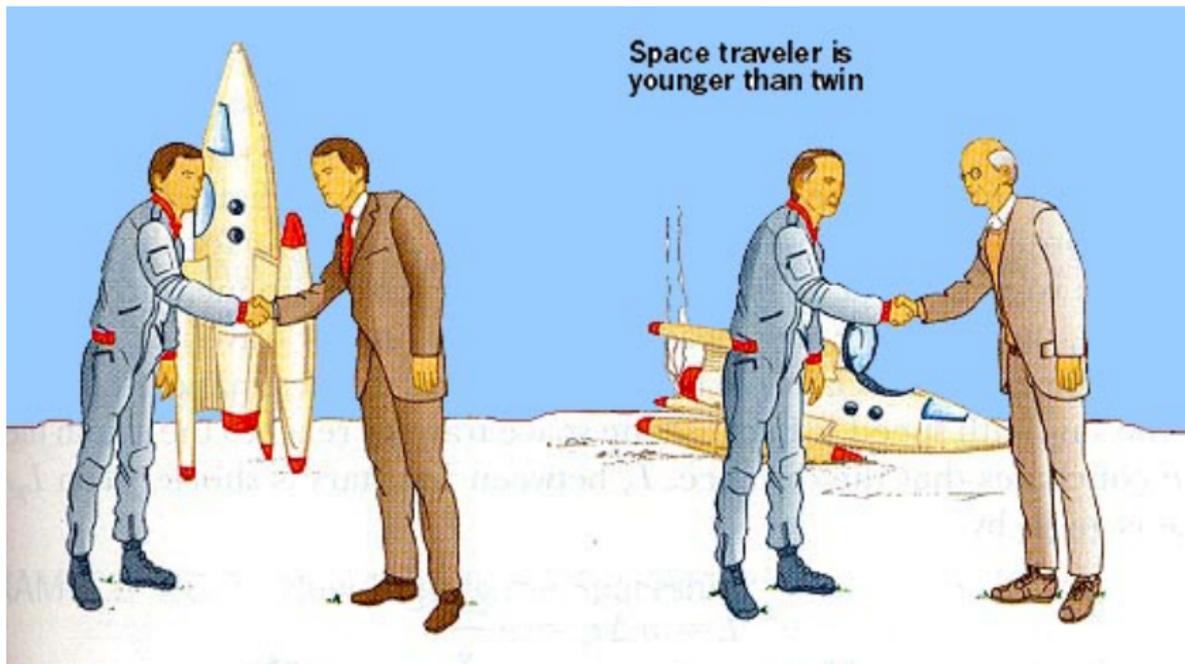
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Speed of light in vacuum same in all inertial systems



The twin problem: an apparent paradox



Time dilation in special relativity

Time dilation

$$\frac{\Delta t'}{\Delta t} = \frac{1}{\sqrt{1 - v^2/c^2}}$$

- ▶ $\Delta t'$: time interval as measured by a moving observer
- ▶ v : the observer's velocity; c : denotes the speed of light
- ▶ Δt : time interval as measured by an observer at rest
- ▶ $1/\sqrt{\quad}$ -term is often called Lorentz factor

Time dilation – how does it effect us?

Time dilation

The speed of light is about $c \approx 299792458$ meters per second.
Let's compare this with the following velocities:

- ▶ speeding car at 100 mph (160 kmh or 44 meter per second)
- ▶ super sonic jet 1000 mph (1600 kmh or 444 meters per second)
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Let's have a look at the numbers

$$\left(\frac{\Delta t'}{\Delta t}\right)_{\text{car}} = 1.000000000000001$$

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Conclusion

Special relativity does not effect our everyday life!

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Special relativity does not effect our everyday life!

Time travel – what can be done?

Into the future

The theory of special relativity, in principle, allows travelling into the future.

Into the past

Travelling into the past does not seem to work, unless time bends into, say, a circle.

Time travel – what can be done?

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Mass-energy equivalence

- ▶ when a flashlight **emits** light, it **loses** mass
- ▶ when a flower **absorbs** sunlight, its mass **increases**
- ▶ when a uranium nucleus splits, the mass of the remnants is **less** than the original mass. The difference appears as **light**, **heat**, and **kinetic energy**

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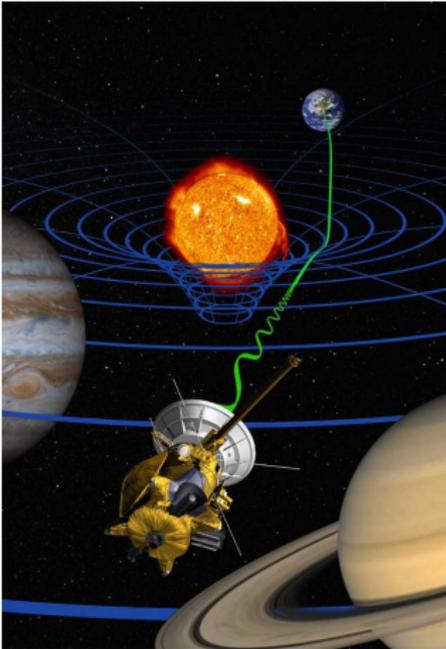
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Nuclear explosion



Matter and the curvature of spacetime



General relativity

- ▶ field equations

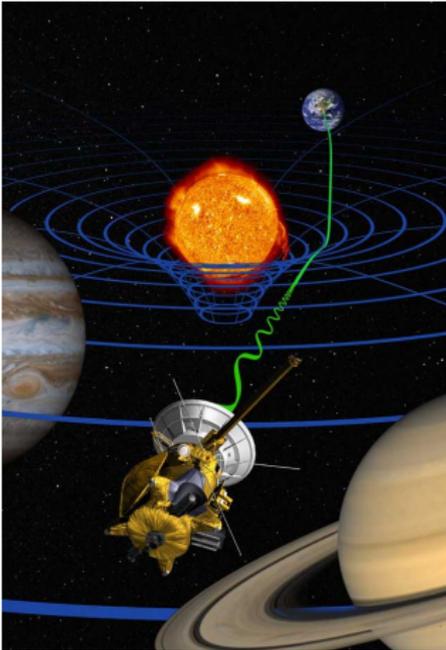
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and

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Matter and the curvature of spacetime



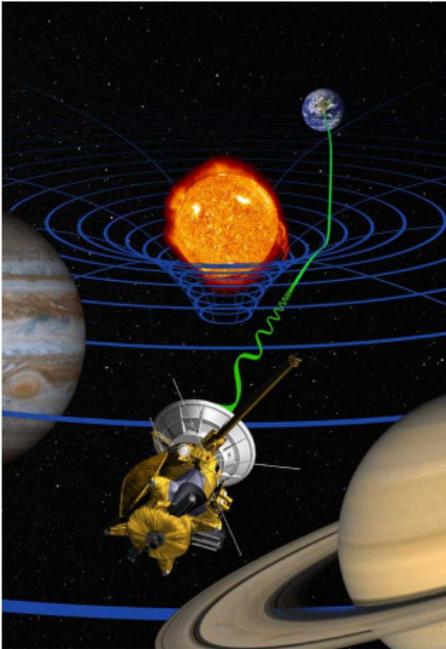
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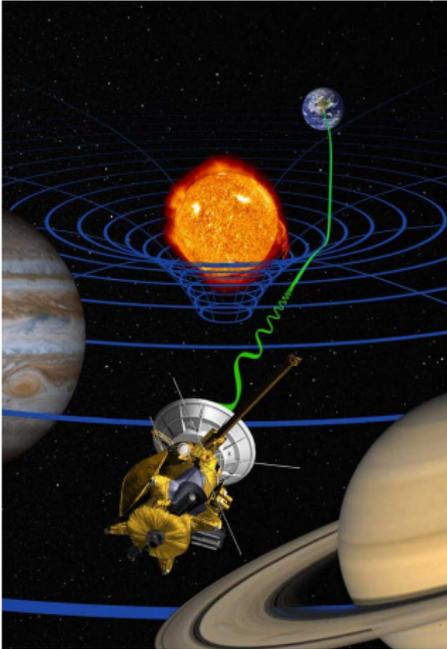
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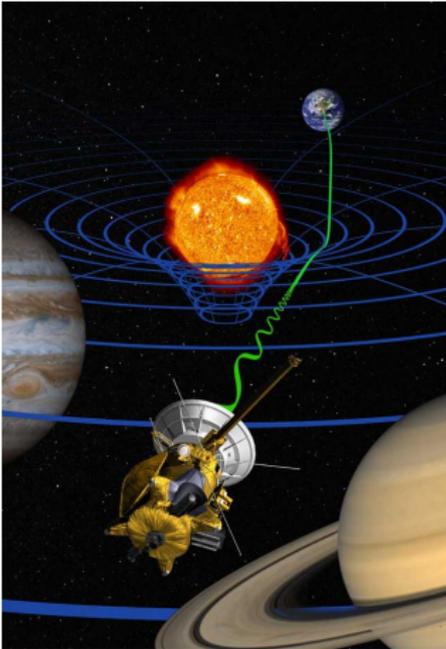
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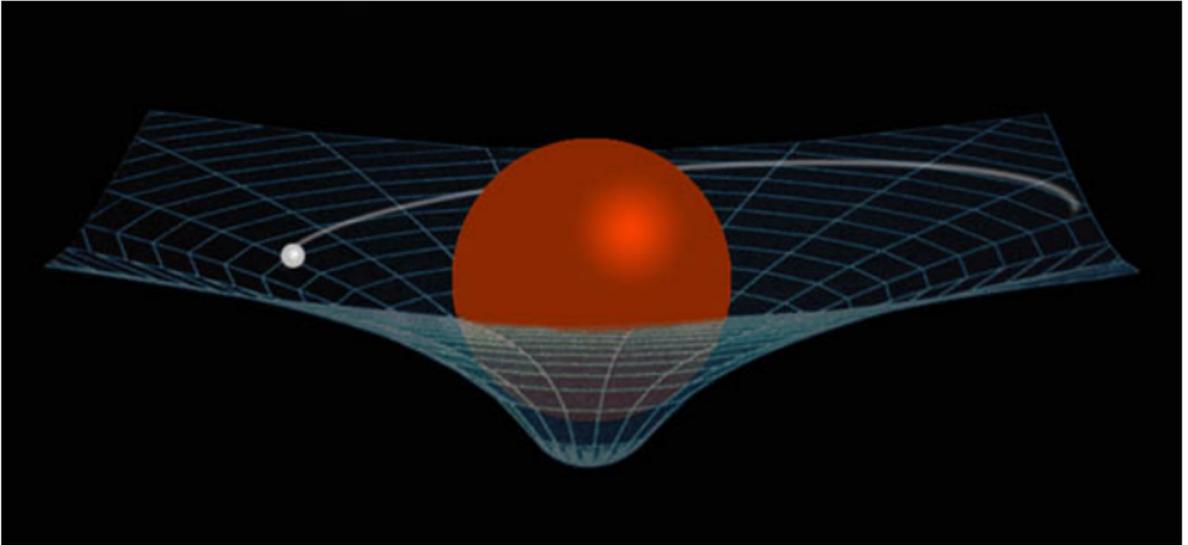
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The motion of a particle in spacetime



Black holes



Are black holes black?

- ▶ black holes are such dense object that not even light can escape their gravitation attraction
- ▶ since they also attract other kind of matter, it is typically very hot near a black hole
- ▶ while the black hole indeed absorbs light, its vicinity is bright

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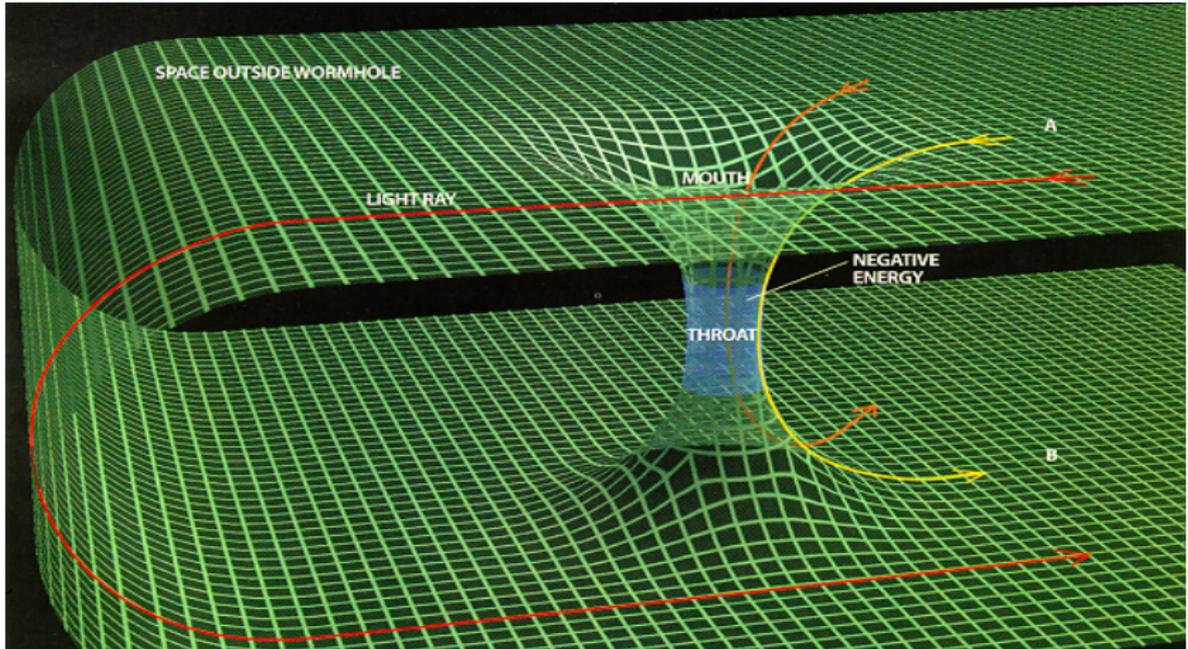
Black holes – a realistic image



Galactic centres

A black hole may be observed by tracking the movement of a group of stars that orbit its centre.

Wormholes in general relativity



Cosmology

What does cosmology mean?

- ▶ cosmology (*κοσμολογια*) – cosmos and logic
- ▶ *κοσμος* – a cosmos is an orderly or harmonious system
- ▶ so cosmology is study of the universe in its totality

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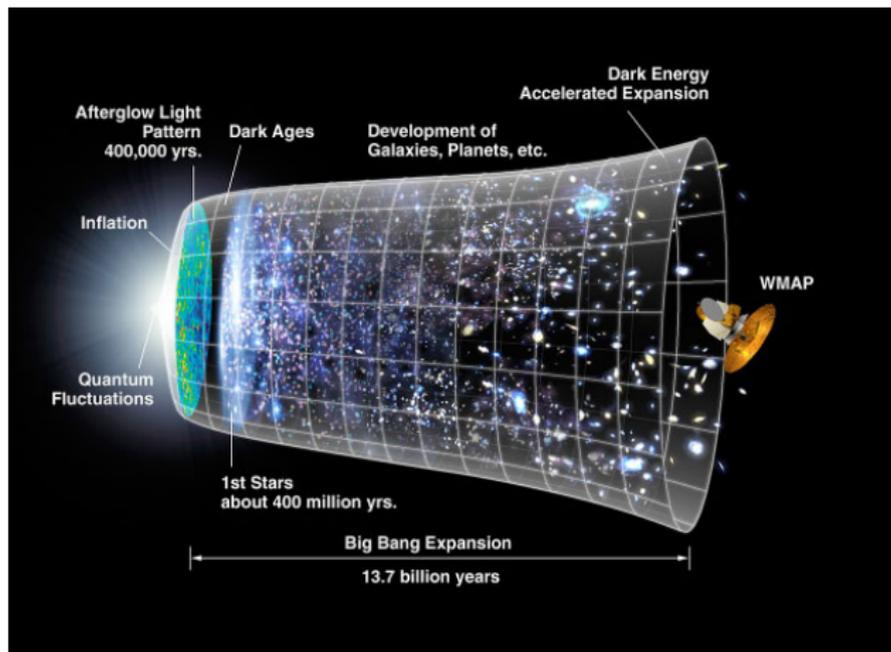
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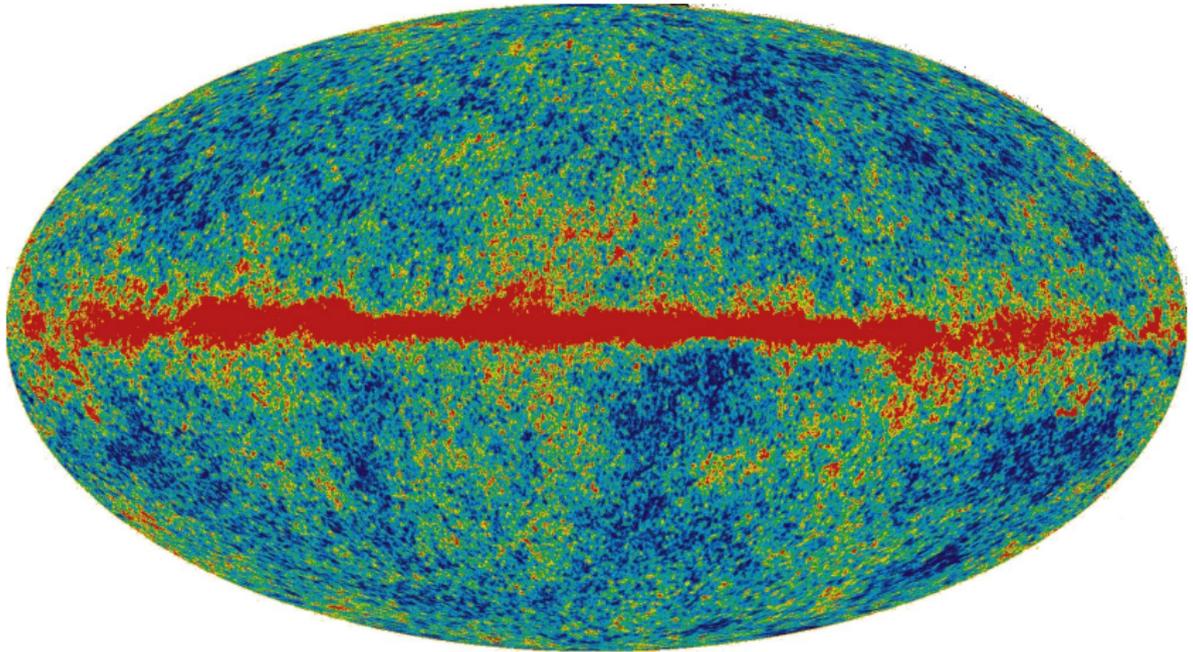
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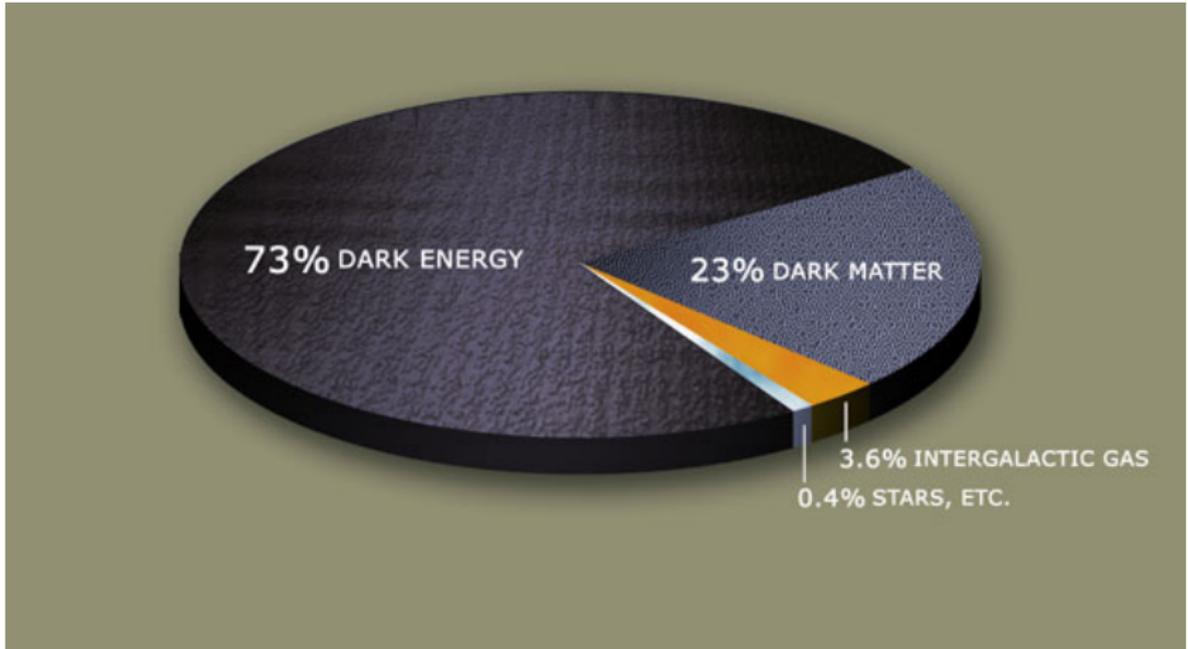
Beyond the big bang



Afterglow light pattern – the cosmic microwave background radiation



The energy content of our Universe



Dark matter



What dark energy is not!



What dark energy could be – the cosmological constant

The cosmological field equations of general relativity

$$R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda g_{ab} = 8\pi T_{ab}$$

properties of spacetime + a cosmological constant = matter

The field equations with cosmological matter

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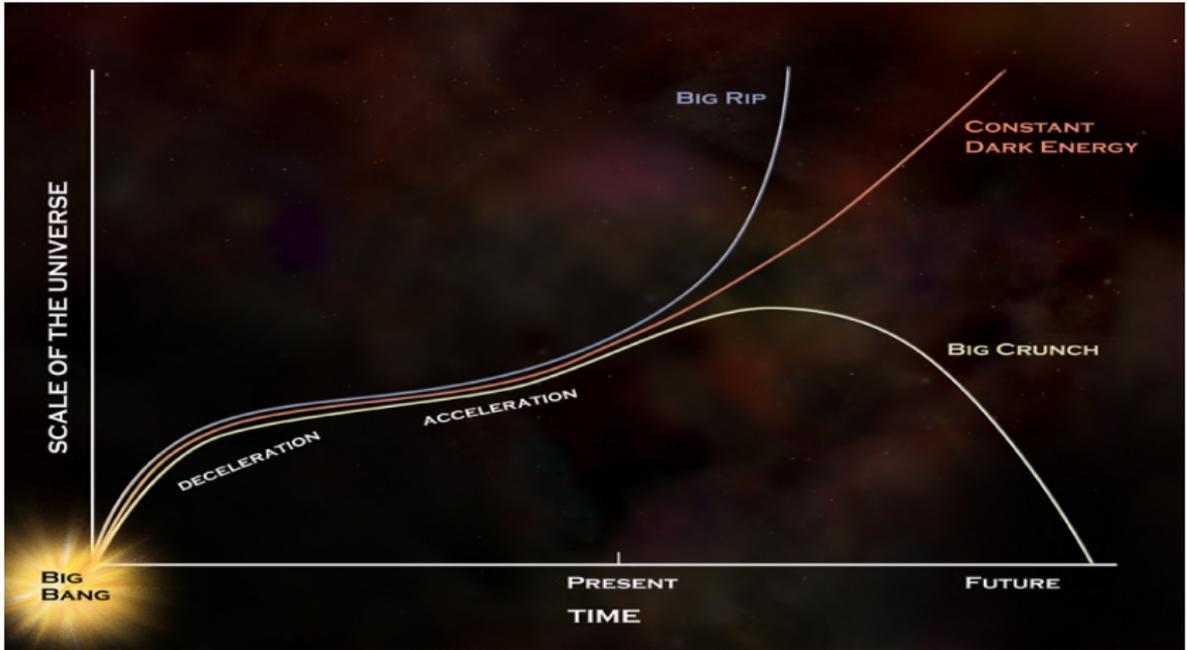
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Possible future of the Universe



Summary

- ▶ Special relativity and general relativity are fascinating subject areas.
- ▶ Special relativity is full of paradoxes that can be solved.
- ▶ General relativity is more or less understood.
- ▶ Cosmology is about 96% non-understood.

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